

DARIEN OLIVA
ECON 453 - PSET 4/5

Question #1

A) Estimate a Regression Model for Medical Expenses

Showing R code and output for the Regression, see script for more detail

```
model1<-lm(medicalexp~income+education+D_RURAL+D_USA+D_CANADA,  
data=pset4_data)
```

PREDICTED MEDICAL EXPENSE =
8.84032+.33494(income)+.50136(education)-12.64338(D_RURAL)

(Since D_USA and D_CANADA have insignificant p-values, they are removed from the model.)

#Residual standard error: 6.891 on 79 degrees of freedom

#Multiple R-squared: 0.7787, Adjusted R-squared: 0.7646

#F-statistic: 55.58 on 5 and 79 DF, p-value:<2.2e-16

The p-value is less than the 1% level of significance, telling us that the data is significant.

```
> print(vcov(model1),digits=4)
```

	(Intercept)	income	education	D_RURAL	D_USA	D_CANADA
(Intercept)	6.9387	-0.022097	-0.322000	-2.80130	-1.18160	-1.06601
income	-0.0221	0.002913	-0.007561	0.01437	-0.02763	-0.02222
education	-0.3220	-0.007561	0.052133	0.10664	0.03339	0.01749
D_RURAL	-2.8013	0.014370	0.106644	3.01003	-0.44412	-0.73254
D_USA	-1.1816	-0.027634	0.033387	-0.44412	3.79750	2.18407
D_CANADA	-1.0660	-0.022221	0.017486	-0.73254	2.18407	3.78708

NOW FOR THE MODEL FOR MEXICO. AVOID DUMMY VARIABLE TRAP

```
> model2 <- lm(medicalexp~income+education+D_RURAL+D_USA+D_MEXICO,  
data=pset4_data)
```

Residual standard error: 6.891 on 79 degrees of freedom

Multiple R-squared: 0.7787, Adjusted R-squared: 0.7646

F-statistic: 55.58 on 5 and 79 DF, p-value: < 2.2e-16

```
> print(vcov(model2),digits=4)
```

	(Intercept)	income	education	D_RURAL	D_USA	D_MEXICO
(Intercept)	8.59374	-0.044318	-0.304514	-3.53384	-1.718607	-2.72107
income	-0.04432	0.002913	-0.007561	0.01437	-0.005413	0.02222
education	-0.30451	-0.007561	0.052133	0.10664	0.015901	-0.01749
D_RURAL	-3.53384	0.014370	0.106644	3.01003	0.288416	0.73254
D_USA	-1.71861	-0.005413	0.015901	0.28842	3.216439	1.60301
D_MEXICO	-2.72107	0.022221	-0.017486	0.73254	1.603012	3.78708

B) Test the null hypothesis that country and location jointly have no effect on medical expenses.

SEE ALL REGRESSIONS FOR Q1B IN ADDENDUM

For this question I ran regressions for medical scores with all countries and locations. For each regression, the result was that the p-value was less than alpha at at least a 5% confidence level. From these results, we can conclude that country and location have a joint effect on medical expenses. Since we could confirm this for each country and location at at least a 5% significance level, we know that the data/relationship is significant, and we must reject the null hypothesis.

C) Construct a 95% Confidence Interval for the Two Households

SEE CODE IN ADDENDUM

The 95% confidence interval for medical expenses for a rural Mexican household with an income of \$50,000 and 12 years of education is between 18.96027 and 33.25869. The 95% confidence interval for medical expenses for an urban US household with the same level of income and education is between 30.03268 and 44.06091. From these intervals we can see that these overlap. The difference in costs for these models is approximately \$11 higher for the USA urban model. The distribution is right shifting.

Question #2

A) Do both schools have increases in scores from 2014-2016?

$$score_{it} = \beta_0 + \beta_1 d_{t=2016} + \epsilon_{it}$$

SEE CODE/FULL REGRESSION RESULTS IN ADDENDUM

$$H_0 : \beta_1 = 0, H_1 : \beta_1 > 0$$

The first regression for the Kennedy School had a p-values less than alpha at at least 5% significance levels, meaning we could reject the null and conclude there was an increase in scores for the Kennedy School between 2014 and 2016. Next I repeated these steps for the Lincoln School, which also had a p-values less than alpha at at least 5% significance levels. This allows us to conclude that the claim is correct that both schools experienced increases in scores between 2014 and 2016.

B) Does performance of fourth graders differ between schools?

SEE CODE/FULL REGRESSION RESULTS IN ADDENDUM

$$score_i = \beta_0 + \beta_1 d + \epsilon_i$$

$$H_0 : \beta_1 = 0, H_1 : \beta_1 \neq 0$$

The regression for testing the differences in scores between schools resulted in a p-value less than alpha at a 5% significance level. It showed that the difference in the data between schools is significant. This allows us to reject the null and conclude that the performance of fourth graders did differ between the Kennedy School and the Lincoln School.

C) Does performance of fourth graders differ between schools each year?

SEE CODE/FULL REGRESSION RESULTS IN ADDENDUM

$$score_i = \beta_0 + \beta_1 d + \beta_2 d^2 + \epsilon_i$$

$$H_0 : \beta_1 = 0, \beta_2 = 0, H_1 : \text{At least one of } \beta_1 \text{ and } \beta_2 \neq 0$$

This regression also resulted in p-values at at least 5% of significance level. This shows us that the difference in data between scores and schools are significantly different in both years 2014 and 2016. Therefore we can reject the null hypothesis and conclude the average test scores for both schools are not the same for either year.

Question #3

A) Scatterplot for the Data (Generated in R)

From the scatterplot below, we can see that the living wage increases with age until approximately the 50 year mark. After the 50 year age mark the living wage begins to decrease as age continues to rise.



B) Data Summary Tables (EXCEL)

TRAINING SET (OBSERVATIONS 1:140)	MODEL 1
ESTIMATED COEFFICIENTS	-
INTERCEPT	40.14946
GRADUATE	6.1061
AGE	0.07249
STANDARD ERRORS	-
INTERCEPT	1.36209
GRADUATE	0.78197
AGE	0.02612
P-VALUES	-
INTERCEPT	2.00E-16
GRADUATE	1.35E-12
AGE	0.00628
R-SQUARED	0.3348
ADJUSTED R-SQUARED	0.325

VALIDATION SET (OBSERVATIONS 140:160)	MODEL 2
ESTIMATED COEFFICIENTS	-
INTERCEPT	-1.018002
GRADUATE	6.078027
AGE	1.856074
I(AGE^2)	-0.018106
STANDARD ERRORS	-
INTERCEPT	5.105533
GRADUATE	1.05605
AGE	0.240329
I(AGE^2)	0.002702
P-VALUES	-
INTERCEPT	8.44E-01
GRADUATE	2.95E-05
AGE	8.73E+00
I(AGE^2)	5.09E-06
R-SQUARED	0.8969
ADJUSTED R-SQUARED	0.8775

C) Predict Expected Wage from Both Models.

FOR MODEL 1 : `predict.lm(model3_1, data.frame(Graduate=1, Age=30))`

For model 1, the estimated wage for a 30-year-old individual with a graduate degree is 48.43022.

FOR MODEL 2 : `predict.lm(model3_2, data.frame(Graduate=1, Age=30))`

For model 2, the estimated wage for a 30-year-old individual with a graduate degree is 44.44729.

D) At What Age Are Wages Maximum?

R CALCULATIONS:

```
> res3_2=summary(model3_2)
```

```
> coef(res3_2)
```

```
      Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.01800194 5.105533196 -0.1993919 8.444681e-01
Graduate     6.07802700 1.056049991  5.7554349 2.952426e-05
Age          1.85607421 0.240328578  7.7230691 8.731945e-07
I(Age^2)     -0.01810551 0.002701852 -6.7011464 5.090307e-06
> -coef(res3_2)[3,1]/(2*coef(res3_2)[4,1])
```

```
[1] 51.25717
```

RESULTS: According to our model, the age in which the living wages are maximized is approximately 51 years old.

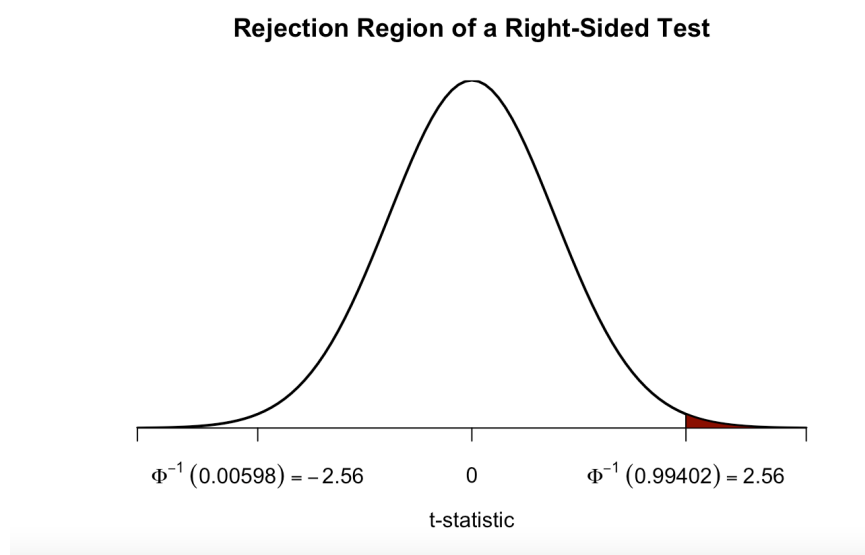
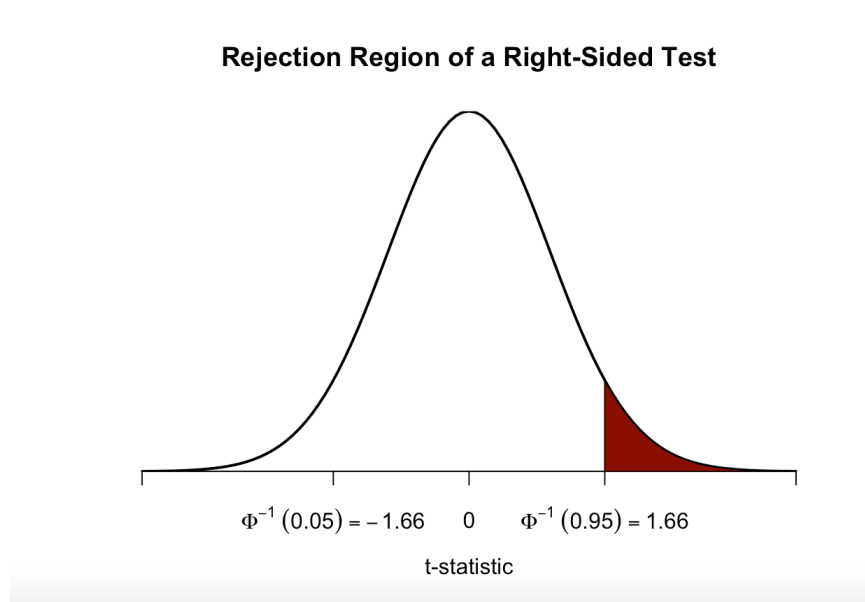
E) Cross Validation Results

DARIEN OLIVA - ECON 453 - PSET4/5		
Cross Validation Results for Wage Data		
TRAINING SET (OBSERVATIONS 1-140)	MODEL 1	MODEL 2
R-SQUARED	0.3348	
ADJUSTED R-SQUARED	0.325	
[correlation (wages, wages)^2]	0.3347565	
MSE	20.10457	
RMSE	4.483812	
MAE	3.664206	
MAPE	8.092382	
RSE	4.532639	
VALIDATION SET (OBSERVATIONS 141-160)		
R-SQUARED		0.8969
ADJUSTED R-SQUARED		0.8775
[correlation (wages, wages)^2]		0.8968757
MSE		2.830505
RMSE		1.682411
MAE		1.385413
MAPE		2.924267
RSE		1.880992

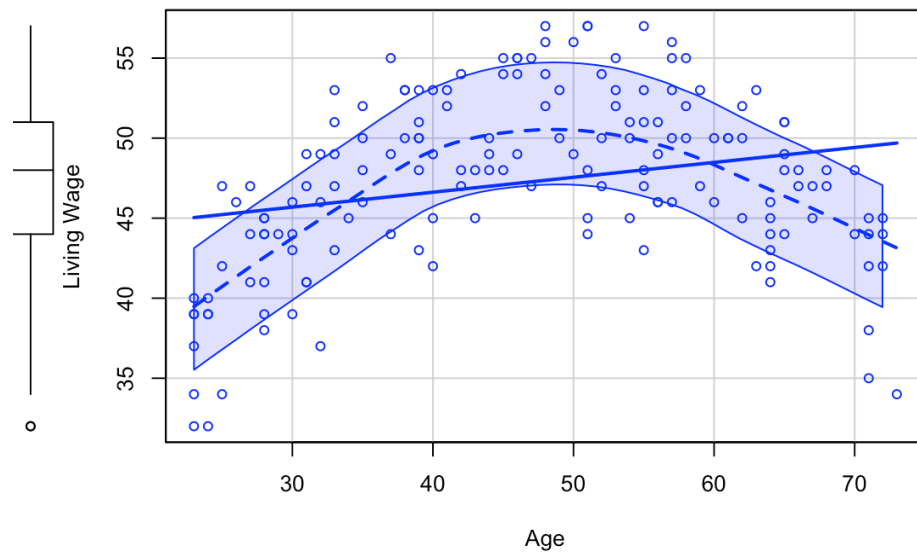
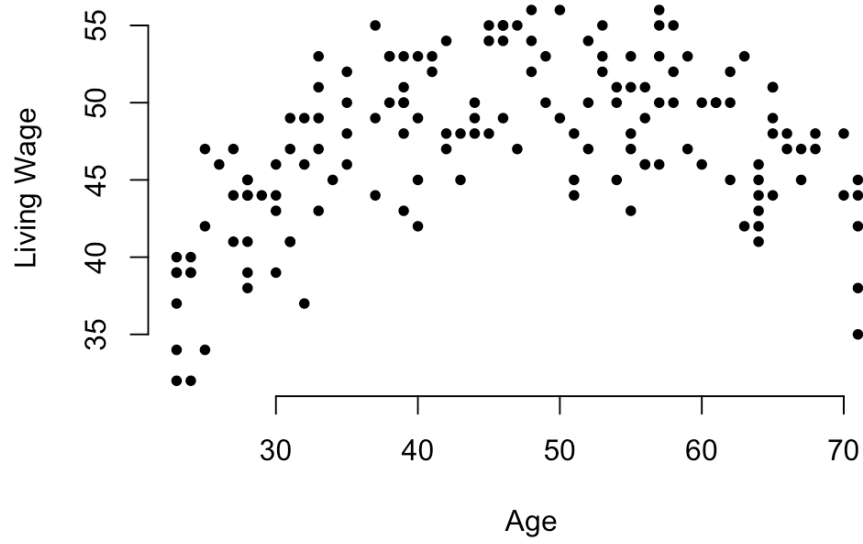
We know that the lower the MAE, MSE, and RMSE, the more accurate the regression model is. Our cross validation shows that all three of the MAE, MSE, and RMSE are lower for the

validation model/model 2. Additionally, the model with the higher R-Squared value is typically preferred and that model is also model 2.

ADDENDUM - R GENERATED PLOTS BY DARIEN OLIVA



Living Wage Pertaining to Age



DARIEN OLIVA - ECON 453 - PSET4/5 - EXCEL TABLES		
TRAINING SET (OBSERVATIONS 1:140)	MODEL 1	
ESTIMATED COEFFICIENTS	-	
INTERCEPT	40.14946	
GRADUATE	6.1061	
AGE	0.07249	
STANDARD ERRORS	-	
INTERCEPT	1.36209	
GRADUATE	0.78197	
AGE	0.02612	
P-VALUES	-	
INTERCEPT	2.00E-16	
GRADUATE	1.35E-12	
AGE	0.00628	
R-SQUARED	0.3348	
ADJUSTED R-SQUARED	0.325	
Cross Validation Results for Wage Data		
TRAINING SET (OBSERVATIONS 1-140)	MODEL 1	MODEL 2
R-SQUARED	0.3348	
ADJUSTED R-SQUARED	0.325	
[correlation (wages, wages)^2]	0.3347565	
MSE	20.10457	
RMSE	4.483812	
MAE	3.664206	
MAPE	8.092382	
RSE	4.532639	
VALIDATION SET (OBSERVATIONS 141-160)		
R-SQUARED		0.8969
ADJUSTED R-SQUARED		0.8775
[correlation (wages, wages)^2]		0.8968757
MSE		2.830505
RMSE		1.682411
MAE		1.385413
MAPE		2.924267
RSE		1.880992
VALIDATION SET (OBSERVATIONS 140:160)	MODEL 2	
ESTIMATED COEFFICIENTS	-	
INTERCEPT	-1.018002	
GRADUATE	6.078027	
AGE	1.856074	
I(AGE^2)	-0.018106	
STANDARD ERRORS	-	
INTERCEPT	5.105533	
GRADUATE	1.05605	
AGE	0.240329	
I(AGE^2)	0.002702	
P-VALUES	-	
INTERCEPT	8.44E-01	
GRADUATE	2.95E-05	
AGE	8.73E+00	
I(AGE^2)	5.09E-06	
R-SQUARED	0.8969	
ADJUSTED R-SQUARED	0.8775	

ADDENDUM / R- SCRIPT BY DARIEN OLIVA

```
> # DARIEN OLIVA - ECON 453 - PSET4&5
> # DATA FROM PSET1
>
> rm(list = ls())
> library(car)
> library(carData)
> library(readxl)
> pset4_data <- read_excel("Downloads/pset1(4&5)_data.xlsx",
+ sheet = "medical_expenses")
> View(pset4_data)
>
> #VIEW DATA STRUCTURE
```

```

> str(pset4_data)
tibble [85 × 5] (S3: tbl_df/tbl/data.frame)
 $ country   : chr [1:85] "USA" "USA" "USA" "USA" ...
 $ location  : chr [1:85] "RURAL" "RURAL" "RURAL" "RURAL" ...
 $ medicalexpnum : num [1:85] 22.27 8.98 8.06 15.6 7.99 ...
 $ income    : num [1:85] 19 42 54 14 36 39 21 75 22 16 ...
 $ education  : num [1:85] 12 10 11 5 11 10 5 14 6 13 ...
>
> # QUESTION 1 A
>
>
>
> #ASSIGN DUMMY VARIABLES, I DISCLUDED MEXICO.
> pset4_data$D_RURAL=ifelse((pset4_data$location == "RURAL"), 1,0)
> pset4_data$D_USA=ifelse((pset4_data$country == "USA"), 1,0)
> pset4_data$D_CANADA=ifelse((pset4_data$country == "CANADA"), 1,0)
> pset4_data$D_MEXICO=ifelse((pset4_data$country == "MEXICO"), 1,0)
>
> #CHECK FOR 1S AND 2S
> pset4_data$Check<-
pset4_data$D_RURAL+pset4_data$D_USA+pset4_data$D_CANADA+pset4_data$D_MEXICO
> head(pset4_data,10)
# A tibble: 10 × 10
  country location medicalexpnum income education D_RURAL D_USA D_CANADA D_MEXICO
Check
  <chr>   <chr>         <dbl> <dbl>    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
1 USA    RURAL         22.3   19      12    1    1    0    0    2
2 USA    RURAL         8.98   42      10    1    1    0    0    2
3 USA    RURAL         8.06   54      11    1    1    0    0    2
4 USA    RURAL        15.6   14       5    1    1    0    0    2
5 USA    RURAL         7.99   36      11    1    1    0    0    2
6 USA    RURAL         5.57   39      10    1    1    0    0    2
7 USA    RURAL          1    21       5    1    1    0    0    2
8 USA    RURAL        25.4   75      14    1    1    0    0    2
9 USA    RURAL         3.43   22       6    1    1    0    0    2
10 USA   RURAL         4.87   16      13    1    1    0    0    2
> tail(pset4_data,5)
# A tibble: 5 × 10
  country location medicalexpnum income education D_RURAL D_USA D_CANADA D_MEXICO
Check
  <chr>   <chr>         <dbl> <dbl>    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
1 MEXICO URBAN        39.7   59      14    0    0    0    1    1
2 MEXICO URBAN        17.9   11       5    0    0    0    1    1
3 MEXICO URBAN        29.6   42      13    0    0    0    1    1

```

```

4 MEXICO URBAN      3.31  13    3    0    0    0    1    1
5 MEXICO URBAN     41.3  39    18   0    0    0    1    1

```

```
>
```

```
> #NEW DATA SUMMARY
```

```
> summary(pset4_data)
```

```

country      location      medicalexp  income      education
Length:85    Length:85      Min.   :1.000 Min.   :4.00 Min.   :0.00
Class :character Class :character 1st Qu.: 8.208 1st Qu.:21.00 1st Qu.: 6.00
Mode  :character Mode  :character Median :16.351 Median :34.00 Median :11.00
              Mean  :19.188 Mean  :37.42 Mean  :10.18
              3rd Qu.:26.822 3rd Qu.:48.00 3rd Qu.:13.00
              Max.   :62.231 Max.   :99.00 Max.   :18.00

```

```

D_RURAL      D_USA      D_CANADA      D_MEXICO      Check
Min.   :0.0000 Min.   :0.0000 Min.   :0.0000 Min.   :0.0000 Min.   :1.000
1st Qu.:0.0000 1st Qu.:0.0000 1st Qu.:0.0000 1st Qu.:0.0000 1st Qu.:1.000
Median :1.0000 Median :0.0000 Median :0.0000 Median :0.0000 Median :2.000
Mean   :0.5294 Mean   :0.3529 Mean   :0.3529 Mean   :0.2941 Mean   :1.529
3rd Qu.:1.0000 3rd Qu.:1.0000 3rd Qu.:1.0000 3rd Qu.:1.0000 3rd Qu.:2.000
Max.   :1.0000 Max.   :1.0000 Max.   :1.0000 Max.   :1.0000 Max.   :2.000

```

```
>
```

```
> #FIRST LINEAR REGRESSION MODEL:
```

```
>
```

```
> model1<-lm(medicalexp~income+education+D_RURAL+D_USA+D_CANADA,
data=pset4_data)
```

```
> summary(model1)
```

Call:

```
lm(formula = medicalexp ~ income + education + D_RURAL + D_USA +
    D_CANADA, data = pset4_data)
```

Residuals:

```

Min      1Q  Median      3Q      Max
-11.392 -4.553 -1.285  4.045 15.259

```

Coefficients:

```

              Estimate Std. Error t value Pr(>|t|)
(Intercept)  8.84034    2.63414   3.356 0.00122 **
income       0.33494    0.05397   6.205 2.36e-08 ***
education    0.50136    0.22833   2.196 0.03104 *
D_RURAL     -12.64338    1.73494  -7.287 2.10e-10 ***
D_USA       -1.57098    1.94872  -0.806 0.42257
D_CANADA    -0.11764    1.94604  -0.060 0.95195

```

```
---
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.891 on 79 degrees of freedom
Multiple R-squared: 0.7787, Adjusted R-squared: 0.7646
F-statistic: 55.58 on 5 and 79 DF, p-value: < 2.2e-16

```
>
> # REGRESSION MODEL/ EQUATION FOR Q#1A
>
> # PREDICTED MEDICAL EXPENSE =
8.84032+.33494(income)+.50136(education)-12.64338(D_RURAL)
> # SINCE D_USA AND D_CANADA HAVE INSIGNIFICANT P-VALUES, THEY ARE
REMOVED FROM THE MODEL.
>
> #Residual standard error: 6.891 on 79 degrees of freedom
> #Multiple R-squared: 0.7787, Adjusted R-squared: 0.7646
> #F-statistic: 55.58 on 5 and 79 DF, p-value:<2.2e-16
> # The p-value is less than the 1% level of significance, telling us that the data is significant.
>
> print(vcov(model1),digits=4)
      (Intercept)  income education  D_RURAL  D_USA D_CANADA
(Intercept)   6.9387 -0.022097 -0.322000 -2.80130 -1.18160 -1.06601
income         -0.0221  0.002913 -0.007561  0.01437 -0.02763 -0.02222
education      -0.3220 -0.007561  0.052133  0.10664  0.03339  0.01749
D_RURAL        -2.8013  0.014370  0.106644  3.01003 -0.44412 -0.73254
D_USA          -1.1816 -0.027634  0.033387 -0.44412  3.79750  2.18407
D_CANADA       -1.0660 -0.022221  0.017486 -0.73254  2.18407  3.78708
>
> # NOW FOR THE MODEL FOR MEXICO. AVOID DUMMY VARIABLE TRAP
> model2 <- lm(medicalexpn~income+education+D_RURAL+D_USA+D_MEXICO,
data=pset4_data)
> summary(model2)
```

Call:

```
lm(formula = medicalexpn ~ income + education + D_RURAL + D_USA +
    D_MEXICO, data = pset4_data)
```

Residuals:

Min	1Q	Median	3Q	Max
-11.392	-4.553	-1.285	4.045	15.259

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	8.72271	2.93151	2.976	0.00388 **
income	0.33494	0.05397	6.205	2.36e-08 ***

```
education    0.50136  0.22833  2.196 0.03104 *
D_RURAL     -12.64338  1.73494 -7.287 2.10e-10 ***
D_USA       -1.45334  1.79344 -0.810 0.42017
D_MEXICO     0.11764  1.94604  0.060 0.95195
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.891 on 79 degrees of freedom
Multiple R-squared: 0.7787, Adjusted R-squared: 0.7646
F-statistic: 55.58 on 5 and 79 DF, p-value: < 2.2e-16

>

```
> print(vcov(model2),digits=4)
```

```
      (Intercept)  income education D_RURAL  D_USA D_MEXICO
(Intercept)    8.59374 -0.044318 -0.304514 -3.53384 -1.718607 -2.72107
income          -0.04432  0.002913 -0.007561  0.01437 -0.005413  0.02222
education       -0.30451 -0.007561  0.052133  0.10664  0.015901 -0.01749
D_RURAL         -3.53384  0.014370  0.106644  3.01003  0.288416  0.73254
D_USA           -1.71861 -0.005413  0.015901  0.28842  3.216439  1.60301
D_MEXICO        -2.72107  0.022221 -0.017486  0.73254  1.603012  3.78708
```

>

```
> # QUESTION 1 B
```

```
> #LINEAR HYPOTHESIS FOR MODEL 2
```

```
> linearHypothesis(model2, c("D_RURAL = 0", "D_USA=0","D_MEXICO=0"))
```

Linear hypothesis test

Hypothesis:

D_RURAL = 0

D_USA = 0

D_MEXICO = 0

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_MEXICO

```
Res.Df  RSS Df Sum of Sq    F  Pr(>F)
1    82 6454.6
2    79 3750.9  3   2703.7 18.981 2.301e-09 ***
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

```
> # P-VALUE IS 0.000000002301 AND LESS THAN THE ALPHA. REJECT THE NULL
HYPOTHESIS AT 5% SIGNIFICANCE LEVEL.
```

>

```
> # LINEAR HYPOTHESIS FOR MODEL 1
```

```
> linearHypothesis(model1, c("D_RURAL = 0", "D_USA=0","D_CANADA=0"))
Linear hypothesis test
```

Hypothesis:

D_RURAL = 0

D_USA = 0

D_CANADA = 0

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_CANADA

```
      Res.Df  RSS Df Sum of Sq    F    Pr(>F)
1       82 6454.6
2       79 3750.9  3    2703.7 18.981 2.301e-09 ***
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

> # P-VALUE IS 0.000000002301 AND LESS THAN ALPHA SO WE CAN REJECT THE NULL HYPOTHEIS

> # AT EITHER 1 OR 5 PERCENT SIGNIFICANCE LEVEL.

> # RESULTS: LIVING IN MEXICO OR IN AN URBAN LOCATON DOES HAVE AN EFFECT ON MEDICAL EXPENSES.

>

> # LINEAR HYPOTHESIS FOR REFERENCE GROUP AND RURAL 1

>

```
> linearHypothesis(model1, c("D_RURAL = 1", "D_USA=0","D_CANADA=0"))
```

Linear hypothesis test

Hypothesis:

D_RURAL = 1

D_USA = 0

D_CANADA = 0

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_CANADA

```
      Res.Df  RSS Df Sum of Sq    F    Pr(>F)
1       82 6890.9
2       79 3750.9  3    3140 22.044 1.807e-10 ***
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

> # P-VALUE IS .0000000001807 WHICH IS LESS THAN ALPHA AT 1% SIGNIFICANCE LEVEL.

```
> # REJECT THE NULL
> # RESULTS: LIVING IN MEXICO OR IN AN URBAN LOCATION DOES HAVE AN EFFECT
ON MEDICAL EXPENSES.
>
> #LINEAR HYPOTHESIS FOR USA AND RURAL
> linearHypothesis(model1, c("D_RURAL = 1", "D_USA=1","D_CANADA=0"))
Linear hypothesis test
```

Hypothesis:
D_RURAL = 1
D_USA = 1
D_CANADA = 0

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_CANADA

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	82	6969.8				
2	79	3750.9	3	3218.8	22.598	1.161e-10 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
>
> # P_VALUE IS LESS THAN 1% SIGNIFICANCE LEVEL. REJECT THE NULL.
> # RESULTS; USA AND RURAL DOES HAVE AN EFFECT ON MEDICAL EXPENSES.
>
> # LINEAR HYPOTHESIS FOR USA AND URBAN
>
> linearHypothesis(model1, c("D_RURAL = 0", "D_USA=1","D_CANADA=0"))
Linear hypothesis test
```

Hypothesis:
D_RURAL = 0
D_USA = 1
D_CANADA = 0

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_CANADA

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	82	6533.1				
2	79	3750.9	3	2782.2	19.532	1.438e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> # P-VALUE IS LESS THAN ALPHA AT 1% SIGNIFICANCE LEVEL, REJECT THE NULL.
> # USA AND URBAN DOES HAVE AN EFFECT ON MEDICAL EXPENSES
>
> # LINEAR HYPOTHESIS FOR CANADA AND RURAL
>
> linearHypothesis(model1, c("D_RURAL = 1", "D_USA=0", "D_CANADA=1"))
Linear hypothesis test
```

Hypothesis:
D_RURAL = 1
D_USA = 0
D_CANADA = 1

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_CANADA

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	82	6966.2				
2	79	3750.9	3	3215.2	22.572	1.184e-10 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> # P-VALUE IS LESS THAN ALPHA AT 1% SIGNIFICANCE LEVEL, REJECT THE NULL.
> # CANADA AND RURAL DOES HAVE AN EFFECT ON MEDICAL EXPENSES
>
> # LINEAR HYPOTHESIS FOR CANADA AND URBAN
>
> linearHypothesis(model1, c("D_RURAL = 0", "D_USA=0", "D_CANADA=1"))
Linear hypothesis test
```

Hypothesis:
D_RURAL = 0
D_USA = 0
D_CANADA = 1

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_CANADA

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	82	6523.6				
2	79	3750.9	3	2772.7	19.465	1.522e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> # P-VALUE IS LESS THAN ALPHA AT 1% SIGNIFICANCE LEVEL, REJECT THE NULL.
> # CANADA AND URBAN DOES HAVE AN EFFECT ON MEDICAL EXPENSES
```

```
>
> #DOUBLE CHECK AGAINST REFERENCE GROUP
> # (MEXICO AND URBAN AREA WITH MODEL #2)
> linearHypothesis(model2, c("D_RURAL = 0", "D_USA=0","D_MEXICO=1"))
Linear hypothesis test
```

Hypothesis:
D_RURAL = 0
D_USA = 0
D_MEXICO = 1

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_MEXICO

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	82	6361.7				
2	79	3750.9	3	2610.7	18.328	4.041e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
>
> #DOUBLE CHECK AGAINST REFERENCE GROUP
> # (MEXICO AND RURAL AREA WITH MODEL #2)
>
> linearHypothesis(model2, c("D_RURAL=1", "D_USA=0","D_MEXICO=1"))
Linear hypothesis test
```

Hypothesis:
D_RURAL = 1
D_USA = 0
D_MEXICO = 1

Model 1: restricted model

Model 2: medicalexpn ~ income + education + D_RURAL + D_USA + D_MEXICO

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	82	6791.5				
2	79	3750.9	3	3040.5	21.346	3.183e-10 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
>
> # QUESTION 1 C
> # CONSTRUCT A 95% CONFIDENCE INTERVAL
>
```

```

> predict.lm(model1, data.frame(income=50, education=12, D_RURAL=1, D_USA=0,
D_CANADA=0),
+           interval="predict", level = 0.95)
      fit    lwr    upr
1 18.96027 4.661852 33.25869
>
> # CHECK THIS WITH MODEL 2 FOR MEXICO AND RURAL
>
> predict.lm(model2, data.frame(income=50, education=12, D_RURAL=1, D_USA=0,
D_MEXICO=1))
      1
18.96027
>
> # IDENTICAL TO MODEL 1
>
> predict.lm(model1, data.frame(income=50, education=12, D_RURAL=0, D_USA=1,
D_CANADA=0),
+           interval="predict", level = 0.95)
      fit    lwr    upr
1 30.03268 16.00445 44.06091
>
> # CHECK THIS WITH MODEL 2 FOR USA AND URBAN
> predict.lm(model2, data.frame(income=50, education=12, D_RURAL=0, D_USA=1,
D_MEXICO=0))
      1
30.03268
>
> # IDENTICAL TO MODEL 1
>
> # COMPARE THE PREDICTED VALUES
>
> PREDICTEDMEXICORURAL = 18.96027
> PREDICTEDUSAURBAN = 30.03268
>
> (PREDICTEDMEXICORURAL)-(PREDICTEDUSAURBAN)
[1] -11.07241
>
> # THE DIFFERENCE IN COSTS FOR THESE MODELS IS APPROXIMATELY $11 FOR THE
USA URBAN MODEL.
> # OUR 95% CONFIDENCE INTERVAL SHOWS THAT THE INTERVALS FOR THE MODELS
DO OVERLAP. RIGHT SHIFTING DISTRIBUTION
>
>
> # QUESTION 2 A

```

```

>
> View(pset4_data)
> library(readxl)
> pset4_data <- read_excel("Downloads/pset1(4&5)_data.xlsx", sheet = "scores")
> View(pset4_data)
> str(pset4_data)
tibble [200 × 3] (S3: tbl_df/tbl/data.frame)
 $ year : num [1:200] 2014 2014 2014 2014 2014 ...
 $ school: chr [1:200] "Lincoln" "Lincoln" "Lincoln" "Lincoln" ...
 $ score : num [1:200] 40 12 88 64 94 12 84 40 94 73 ...
>
>
> # ASSIGN DUMMY VARIABLES
> # 0 FOR LINCOLN
> # 1 FOR KENNEDY
>
> pset4_data$d_ken=ifelse((pset4_data$school=="Kennedy"), 1, 0)
> pset4_data$d_2016=ifelse((pset4_data$year==2016), 1, 0)
> summary(pset4_data)
   year      school      score      d_ken      d_2016
Min. :2014 Length:200   Min. : 3.00 Min. :0.0 Min. :0.0
1st Qu.:2014 Class :character 1st Qu.: 47.75 1st Qu.:0.0 1st Qu.:0.0
Median :2015 Mode  :character Median : 63.00 Median :0.5 Median :0.5
Mean   :2015          Mean : 62.37 Mean :0.5 Mean :0.5
3rd Qu.:2016          3rd Qu.: 83.00 3rd Qu.:1.0 3rd Qu.:1.0
Max.   :2016          Max. :100.00 Max. :1.0 Max. :1.0
>
> # FIRST LINEAR MODEL FOR KENNEDY
>
> mken16 = lm(data=pset4_data[pset4_data$d_ken==1,],score~ d_2016)
> summary(mken16)

```

Call:

```
lm(formula = score ~ d_2016, data = pset4_data[pset4_data$d_ken ==
1, ])
```

Residuals:

```

Min    1Q  Median    3Q   Max
-31.58 -15.37 -0.27  17.04  37.42

```

Coefficients:

```

      Estimate Std. Error t value Pr(>|t|)
(Intercept)  62.580     2.590  24.161 <2e-16 ***
d_2016        9.380     3.663   2.561  0.012 *

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 18.31 on 98 degrees of freedom

Multiple R-squared: 0.06272, Adjusted R-squared: 0.05315

F-statistic: 6.558 on 1 and 98 DF, p-value: 0.01197

>

> # RESULTS; THE P-VALUE IS LESS THAN THE ALPHA AT 5% SIGNIFICANCE LEVEL.

> # REJECT THE NULL HYPOTHESIS. THE DATA IN THIS MODEL IS SIGNIFICANT.

>

> res<-summary(mken16)

> coef(res)

	Estimate	Std. Error	t value	Pr(> t)
--	----------	------------	---------	----------

(Intercept)	62.58	2.590095	24.161276	4.571907e-43
-------------	-------	----------	-----------	--------------

d_2016	9.38	3.662947	2.560779	1.196947e-02
--------	------	----------	----------	--------------

> pt(coef(res)[2,3], 98)

[1] 0.9940153

> 1-pt(coef(res)[2,3], 98)

[1] 0.005984733

>

> m16 <- lm(score~d_ken+d_2016, data=pset4_data)

> summary(m16)

Call:

lm(formula = score ~ d_ken + d_2016, data = pset4_data)

Residuals:

Min	1Q	Median	3Q	Max
-49.05	-16.10	-0.37	18.04	46.95

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
--	----------	------------	---------	----------

(Intercept)	52.050	2.731	19.056	< 2e-16 ***
-------------	--------	-------	--------	-------------

d_ken	9.800	3.154	3.107	0.002168 **
-------	-------	-------	-------	-------------

d_2016	10.840	3.154	3.437	0.000718 ***
--------	--------	-------	-------	--------------

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 22.3 on 197 degrees of freedom

Multiple R-squared: 0.09826, Adjusted R-squared: 0.08911

F-statistic: 10.73 on 2 and 197 DF, p-value: 3.762e-05

>

```
> # RESULTS; THE P-VALUE IS LESS THAN THE ALPHA AT 1% SIGNIFICANCE LEVEL.
```

```
> linearHypothesis(m16, c("d_ken=1", "d_2016=1"))
```

Linear hypothesis test

Hypothesis:

d_ken = 1

d_2016 = 1

Model 1: restricted model

Model 2: score ~ d_ken + d_2016

```
Res.Df  RSS Df Sum of Sq    F  Pr(>F)
1   199 106697
2   197  97983  2    8713.3 8.7592 0.0002268 ***
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
>
```

```
> # PLOT THE T DISTRIBUTION DENSITY OVER THE DOMAIN [-4,4]
```

```
> curve(dt(x, df=98),
```

```
+   xlim = c(-4, 4),
```

```
+   main = "Rejection Region of a Right-Sided Test", yaxs = "i",
```

```
+   xlab = "t-statistic",
```

```
+   ylab = "",
```

```
+   lwd = 2,
```

```
+   axes = "F")
```

```
> axis(1,
```

```
+   at = c(-4,-qt(0.95, 98), 0, qt(0.95, 98), 4),
```

```
+   padj = 0.5,
```

```
+   labels = c("", expression(Phi^1~(.05)==-1.66), 0, expression(Phi^1~(.95)==1.66), ""))
```

```
> polygon(x = c(1.66, seq(1.66, 4, 0.01), 4),
```

```
+   y = c(0, dt(seq(1.66, 4, 0.01),df=98), 0), col = "darkred")
```

```
>
```

```
> pt(coef(res)[2,3], 98)
```

```
[1] 0.9940153
```

```
> 1-pt(coef(res)[2,3], 98)
```

```
[1] 0.005984733
```

```
> 2*(1-pt(coef(res)[2,3], 98))
```

```
[1] 0.01196947
```

```
>
```

```
> curve(dt(x, df=98),
```

```
+   xlim = c(-4, 4),
```

```
+   main = "Rejection Region of a Right-Sided Test", yaxs = "i",
```

```
+   xlab = "t-statistic",
```

```
+   ylab = "",
```

```

+   lwd = 2,
+   axes = "F")
> axis(1,
+   at = c(-4,-2.560779, 0, 2.560779, 4),
+   padj = 0.5,
+   labels = c("", expression(Phi^-1~(0.00598)==-2.56), 0,
expression(Phi^-1~(0.99402)==2.56),
+   ""))
> polygon(x = c(2.56, seq(2.56, 4, 0.01), 4),
+   y = c(0, dt(seq(2.56, 4, 0.01), df=98),0),
+   col = "darkred")
>
> # FOR LINCOLN 2016
>
> ml16=lm(data=pset4_data[pset4_data$d_ken==0,],score~ d_2016)
> summary(ml16)

```

Call:

```
lm(formula = score ~ d_2016, data = pset4_data[pset4_data$d_ken ==
0, ])
```

Residuals:

Min	1Q	Median	3Q	Max
-48.32	-17.62	-0.62	19.38	47.68

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	51.320	3.642	14.090	<2e-16 ***
d_2016	12.300	5.151	2.388	0.0189 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 25.75 on 98 degrees of freedom

Multiple R-squared: 0.05499, Adjusted R-squared: 0.04534

F-statistic: 5.702 on 1 and 98 DF, p-value: 0.01886

```

>
> # P-VALUE IS LESS THAN ALPHA AT SIGNIIFICANCE LEVEL 5%.
> # REJECT THE NULL HYPOTHESIS
>
> # FOR LINCOLN 2014
>
> linearHypothesis(ml16, c("d_2016 = 0"))
Linear hypothesis test

```

Hypothesis:

$d_{2016} = 0$

Model 1: restricted model

Model 2: $\text{score} \sim d_{2016}$

```
Res.Df  RSS Df Sum of Sq    F Pr(>F)
1    99 68787
2    98 65005  1   3782.3 5.7021 0.01886 *
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

> # P-VALUE IS LESS THAN ALPHA AT 5% SIGNIFICANCE LEVEL

> # REJECT THE NULL. RESULTS: THERE WAS AN EFFECT ON SCORES IN YEAR 2016.

>

> linearHypothesis(ml16, c("d_2016 = 1"))

Linear hypothesis test

Hypothesis:

$d_{2016} = 1$

Model 1: restricted model

Model 2: $\text{score} \sim d_{2016}$

```
Res.Df  RSS Df Sum of Sq    F Pr(>F)
1    99 68197
2    98 65005  1   3192.3 4.8126 0.03062 *
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

> # P-VALUE IS LESS THAN ALPHA AT 5% SIGNIFICANCE LEVEL

> # REJECT THE NULL

>

> m2_3=lm(data=pset4_data[pset4_data\$d_2016==0,],score~ d_ken)

> summary(m2_3)

Call:

lm(formula = $\text{score} \sim d_{\text{ken}}$, data = pset4_data[pset4_data\$d_2016 == 0,])

Residuals:

```
Min      1Q  Median      3Q      Max
-48.32 -18.39   1.42  18.48  47.68
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	51.320	3.596	14.270	<2e-16 ***
d_ken	11.260	5.086	2.214	0.0292 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 25.43 on 98 degrees of freedom

Multiple R-squared: 0.04763, Adjusted R-squared: 0.03791

F-statistic: 4.901 on 1 and 98 DF, p-value: 0.02915

>

> # P-VALUE IS LESS THAN ALPHA AT 5% SIGNIFICANCE LEVEL

> # REJECT THE NULL

>

> linearHypothesis(m2_3, c("d_ken = 0"))

Linear hypothesis test

Hypothesis:

d_ken = 0

Model 1: restricted model

Model 2: score ~ d_ken

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	99	66545				
2	98	63375	1	3169.7	4.9014	0.02915 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

> # P-VALUE IS LESS THAN ALPHA AT 5% SIGNIFICANCE LEVEL

> # REJECT THE NULL

>

> linearHypothesis(m2_3, c("d_ken = 1"))

Linear hypothesis test

Hypothesis:

d_ken = 1

Model 1: restricted model

Model 2: score ~ d_ken

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
--	--------	-----	----	-----------	---	--------

```
1 99 66007
2 98 63375 1 2631.7 4.0695 0.0464 *
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

> # P-VALUE IS LESS THAN ALPHA AT 5% SIGNIFICANCE LEVEL

> # REJECT THE NULL. RESULTS: THE SCHOOL HAS A SIGNIFICANT EFFECT.

>

> m2_4=lm(data=pset4_data[pset4_data\$d_2016==1,],score~ d_ken)

> summary(m2_4)

Call:

lm(formula = score ~ d_ken, data = pset4_data[pset4_data\$d_2016 ==
1,])

Residuals:

Min	1Q	Median	3Q	Max
-40.62	-14.71	-2.29	18.12	34.38

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	63.620	2.654	23.976	<2e-16 ***
d_ken	8.340	3.753	2.222	0.0286 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 18.76 on 98 degrees of freedom

Multiple R-squared: 0.04798, Adjusted R-squared: 0.03827

F-statistic: 4.939 on 1 and 98 DF, p-value: 0.02855

>

> # THE P-VALUE IS 0.02855. THIS IS LESS THAN THE ALPHA AT 5% SIGNIFICANCE LEVEL.

> # REJECT THE NULL.

>

> m2_5=lm(data=pset4_data,score~ d_ken+d_2016)

> summary(m2_5)

Call:

lm(formula = score ~ d_ken + d_2016, data = pset4_data)

Residuals:

Min	1Q	Median	3Q	Max
-49.05	-16.10	-0.37	18.04	46.95

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	52.050	2.731	19.056	< 2e-16 ***
d_ken	9.800	3.154	3.107	0.002168 **
d_2016	10.840	3.154	3.437	0.000718 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 22.3 on 197 degrees of freedom

Multiple R-squared: 0.09826, Adjusted R-squared: 0.08911

F-statistic: 10.73 on 2 and 197 DF, p-value: 3.762e-05

>

> linearHypothesis(m2_5, c("d_ken = 0", "d_2016=0"))

Linear hypothesis test

Hypothesis:

d_ken = 0

d_2016 = 0

Model 1: restricted model

Model 2: score ~ d_ken + d_2016

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	199	108661				
2	197	97983	2	10677	10.734	3.762e-05 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>

> # THE P-VALUE IS 0.00003762. THIS IS LESS THAN THE ALPHA AT 5% SIGNIFICANCE LEVEL.

> # REJECT THE NULL.

>

> linearHypothesis(m2_5, c("d_ken = 1", "d_2016=1"))

Linear hypothesis test

Hypothesis:

d_ken = 1

d_2016 = 1

Model 1: restricted model

Model 2: score ~ d_ken + d_2016

```

    Res.Df  RSS Df Sum of Sq    F  Pr(>F)
1    199 106697
2    197  97983  2    8713.3 8.7592 0.0002268 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>
> # THE P-VALUE IS 0.0002268. THIS IS LESS THAN THE ALPHA AT 5% SIGNIFICANCE
LEVEL.
> # REJECT THE NULL.
>
>
> #QUESTION 3 A
>
> # DRAW SCATTERPLOT
>
> library(readxl)
> chpt08wagedata <- read_excel("Downloads/jaggia_ba_2e_ch08_data.xlsx",
+   sheet = "Wages")
> View(chpt08wagedata)
> str(chpt08wagedata)
tibble [160 × 3] (S3: tbl_df/tbl/data.frame)
 $ Wage   : num [1:160] 53 47 32 43 42 47 41 55 48 46 ...
 $ Graduate: num [1:160] 1 0 0 0 1 0 1 1 1 1 ...
 $ Age    : num [1:160] 57 42 24 39 72 52 27 46 68 35 ...
>
> x<- chpt08wagedata$Age
> y<- chpt08wagedata$Wage
> plot(x, y, main = "Living Wage Pertaining to Age",
+   xlab = "Age", ylab = "Living Wage", pch = 20, frame = FALSE)
>
> # ADD FIT LINES AND REGRESSION
> molog <- lm(y ~ log(x))
> summary(molog)

Call:
lm(formula = y ~ log(x))

Residuals:
    Min     1Q   Median     3Q    Max
-16.0940 -3.5116  0.5128  3.6541  9.2964

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  25.632    4.632   5.533 1.28e-07 ***

```

log(x) 5.702 1.217 4.685 6.00e-06 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.155 on 158 degrees of freedom

Multiple R-squared: 0.122, Adjusted R-squared: 0.1164

F-statistic: 21.95 on 1 and 158 DF, p-value: 6e-06

```
> scatterplot(chpt08wagedata$Wage~chpt08wagedata$Age,
+             xlab = "Age", ylab = "Living Wage")
>
> # THE LIVING WAGE INCREASES WITH AGE UNTIL APPROXIMATELY 50 YEARS OLD.
> # AFTER THE 50 YEAR AGE MARK, THE LIVING WAGE DECREASES AS AGE
INCREASES.
>
> # QUESTION 3 B
>
> # SEPERATE THE DATA FROM 1-140 AND 141-160
>
> chpt08wagedata_T = chpt08wagedata[1:140,];
> chpt08wagedata_v = chpt08wagedata [141:160,];
> model3_1=lm( Wage ~ Graduate+Age,chpt08wagedata_T)
> summary(model3_1)
```

Call:

lm(formula = Wage ~ Graduate + Age, data = chpt08wagedata_T)

Residuals:

Min	1Q	Median	3Q	Max
-11.4411	-2.9854	0.4416	3.8157	7.2650

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	40.14946	1.36209	29.476	< 2e-16 ***
Graduate	6.10610	0.78197	7.809	1.35e-12 ***
Age	0.07249	0.02612	2.776	0.00628 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.533 on 137 degrees of freedom

Multiple R-squared: 0.3348, Adjusted R-squared: 0.325

F-statistic: 34.47 on 2 and 137 DF, p-value: 7.485e-13

>

```

> # P-VALUE IS LESS THAN THE ALPHA AT 5% SIGNIFICANCE LEVEL.
> # REJECT THE NULL HYPOTHEIS
>
> # ADD ANOTHER COLUMN FOR AGE SQUARED AND RUN NEW REGRESSION
> model3_2=lm(data = chpt08wagedata_v, Wage~Graduate+Age+ I(Age^2))
> summary(model3_2)

```

Call:

```
lm(formula = Wage ~ Graduate + Age + I(Age^2), data = chpt08wagedata_v)
```

Residuals:

```

      Min       1Q   Median       3Q      Max
-3.3941 -1.0262  0.0088  1.1640  2.9685

```

Coefficients:

```

              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.018002   5.105533  -0.199   0.844
Graduate      6.078027   1.056050   5.755 2.95e-05 ***
Age           1.856074   0.240329   7.723 8.73e-07 ***
I(Age^2)     -0.018106   0.002702  -6.701 5.09e-06 ***
---

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.881 on 16 degrees of freedom
Multiple R-squared: 0.8969, Adjusted R-squared: 0.8775
F-statistic: 46.38 on 3 and 16 DF, p-value: 4.07e-08

```

>
> # QUESTION 3 C
> # PREDICT THE WAGE FOR A 30 YEAR OLD GRADUATE STUDENT.
> #USING MODEL 3_1 AND MODEL 3_2
>
> # FOR MODEL 3_1
> predict.lm(model3_1, data.frame(Graduate=1, Age=30))
      1
48.43022
> # USING MODEL3_1 THE PREDICTED WAGE FOR A 30 YEAR OLD GRADUATE
STUDENT IS 48.43
>
>
> # FOR MODEL 3_2
> predict.lm(model3_2, data.frame(Graduate=1, Age=30))
      1
44.44729

```

```

> # USING MODEL3_1 THE PREDICTED WAGE FOR A 30 YEAR OLD GRADUATE
STUDENT IS 44.45
>
>
> # QUESTION 3 D
> # AT WHAT AGE ARE WAGES THE MAXIMUM AMOUNT?
>
> res3_2=summary(model3_2)
> coef(res3_2)
      Estimate Std. Error  t value    Pr(>|t|)
(Intercept) -1.01800194  5.105533196 -0.1993919 8.444681e-01
Graduate      6.07802700  1.056049991  5.7554349 2.952426e-05
Age           1.85607421  0.240328578  7.7230691 8.731945e-07
I(Age^2)     -0.01810551  0.002701852 -6.7011464 5.090307e-06
> -coef(res3_2)[3,1]/(2*coef(res3_2)[4,1])
[1] 51.25717
>
> # THE AGE IN WHICH WAGES ARE THE MAXIMUM IS APPROXIMATELY 51 YEARS OLD.
>
>
> # QUESTION 3 E
>
> # Cross validation. Calculate, MSE, RMSE, MAPE, MAE, MAPE
> # Do for both models over the training set and over the validation set.
>
> # FIRST FOR TRAINING SET
>
> res3_1=summary(model3_1)
> coef(res3_1)
      Estimate Std. Error  t value    Pr(>|t|)
(Intercept) 40.14945877  1.36209119 29.476337 3.595770e-61
Graduate     6.10610322  0.78196849  7.808631 1.346943e-12
Age          0.07248865  0.02611639  2.775600 6.280670e-03
>
> chpt08wagedata_T$yhat=predict.lm(model3_1,chpt08wagedata_T)
> chpt08wagedata_T$res3_1=resid(model3_1)
>
> (cor(chpt08wagedata_T$Wage, chpt08wagedata_T$yhat))^2
[1] 0.3347565
>
> MSE3_1=(sum((chpt08wagedata_T$Wage-chpt08wagedata_T$yhat)^2)/(140))
> MSE3_1
[1] 20.10457
>

```

```

> RMSE3_1=MSE3_1^0.5
> RMSE3_1
[1] 4.483812
>
> MAE3_1=mean(abs(chpt08wagedata_T$Wage-chpt08wagedata_T$yhat))
> MAE3_1
[1] 3.664206
>
> MAPE3_1=mean(abs(chpt08wagedata_T$Wage-
chpt08wagedata_T$yhat)/chpt08wagedata_T$Wage*100)
> MAPE3_1
[1] 8.092382
>
> RSE3_1=(sum((chpt08wagedata_T$Wage-chpt08wagedata_T$yhat)^2)/(140-3))^0.5
> RSE3_1
[1] 4.532639
>
> # NOW VALIDATIION SET
>
> res3_2=summary(model3_2)
> coef(res3_2)
      Estimate Std. Error  t value    Pr(>|t|)
(Intercept) -1.01800194  5.105533196 -0.1993919 8.444681e-01
Graduate     6.07802700  1.056049991  5.7554349 2.952426e-05
Age          1.85607421  0.240328578  7.7230691 8.731945e-07
I(Age^2)    -0.01810551  0.002701852 -6.7011464 5.090307e-06
> chpt08wagedata_v$yhat=predict.lm(model3_2,chpt08wagedata_v)
> chpt08wagedata_v$res3_2=resid(model3_2)
> (cor(chpt08wagedata_v$Wage,chpt08wagedata_v$yhat))^2
[1] 0.8968757
>
> MSE3_2=(sum((chpt08wagedata_v$Wage-chpt08wagedata_v$yhat)^2)/(20))
> MSE3_2
[1] 2.830505
>
> RMSE3_2=MSE3_2^0.5
> RMSE3_2
[1] 1.682411
>
> MAE3_2=mean(abs(chpt08wagedata_v$Wage-chpt08wagedata_v$yhat))
> MAE3_2
[1] 1.385413
>

```

```
> MAPE3_2=mean(abs(chpt08wagedata_v$Wage-  
chpt08wagedata_v$yhat)/chpt08wagedata_v$Wage*100)  
> MAPE3_2  
[1] 2.924267  
>  
> RSE3_2=(sum((chpt08wagedata_v$Wage-chpt08wagedata_v$yhat)^2)/(20-4))^0.5  
> RSE3_2  
[1] 1.880992  
>  
> # CREATE TABLE IN EXCEL TO DISPLAY THESE RESULTS.
```